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Center of Gravity and Minimal Lift Coefficient Limits of a Gliding Parachute

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Standard static stability analysis is used to reveal a peculiar nature of longitudinal c.g. limits of a gliding parachute. Specifically, it is shown that the most forward c.g. position of the parachute is limited by a loss of longitudinal static stability and by a loss of control power, whereas the most rear c.g. position is limited by a requirement of stall-free controls range. It is also shown that a forward c.g. limit imposes a limit on minimal lift coefficient possibly attainable at trim. A typical value of this minimal lift coefficient limit predicted by the present analysis is in a good agreement with the top speed data of currently flying designs.

I. Introduction

THE term "gliding parachute" is associated with several types of vessels utilizing ram-air-inflated flexible canopies to produce lift. In this exposition we shall use this term to imply mainly a recreational soaring vessel operated by a single pilot.

Longitudinal and lateral control of a gliding parachute is conventionally done with the lines attached to the trailing edge of the canopy, a pull on these lines causes the trailing part of the canopy to flex downward, serving much the same as a pair of conventional elevons. In some designs, a pilot can also control the lengths of the lines attached to the forward half of the canopy so as to move himself (and the vessel's c.g.) forward and backward relative to the canopy.

In the context of longitudinal control, two flying qualities of a gliding parachute seem peculiar. One is that for a longitudinally stable vessel, the lift coefficient increases with elevons deflected downward. The other is that, judging from the top speed data of currently flying designs, there seems to exist a minimal lift coefficient (roughly 0.5) at which a conventionally built gliding parachute can be possibly trimmed, regardless of the particularities of its design.

In this exposition, a standard longitudinal static stability analysis (e.g., Ref. 1) is used to show that a minimal lift coefficient limit does indeed exist; moreover, it is a consequence of requiring the lift coefficient to be an increasing function of elevons deflection. Specifically, it is shown that, in apparent contrast with a conventional vessel, a loss of longitudinal static stability and a loss of control power each imposes a limit on the most forward c.g. position, and, concurrently, on the minimal lift coefficient possibly attainable at trim. It is also shown that, with a proper design, requiring the lift coefficient to be an increasing function of elevons deflection is sufficient to ensure longitudinal static stability; in which case both the forward c.g. and minimal lift coefficient limits are associated with a loss of control power.

Under several simplifying assumptions, linear lift curve slope, parabolic drag polar, fixed-shape canopy, fixed-lengths lines, and plain flaps for elevons, the present analysis suggests a very simple expression for the minimal lift coefficient limit. The value of this limit depends, mainly, on the relative elevons chord and on the lines-to-chord lengths ratio; specifically, it decreases with an increase in either parameter. A sample calculation of the minimal lift coefficient limit for a typical gliding parachute yields, indeed, about 0.5.

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II. Pitching Moment

Consider a gliding parachute in a symmetric unaccelerated flight. Following conventional definitions of aerodynamic coefficients, let the projected wing area and its mean aerodynamic chord serve as the respective references. Let $C_{d,w}$, $C_{d,l}$, and $C_{d,p}$ be drag coefficients of the canopy, the lines, and the pilot, respectively. Let, also, C_L and $C_{M,w0}$ be the lift coefficient and the pitching moment coefficient of the canopy about its aerodynamic center, respectively.

Select a standard Cartesian coordinate system, with the x, y, and z axes pointing forward, right and downward, respectively. For the sake of being specific, it will be assumed that the x axis connects the trailing and the leading edges of the midsection of the wing. Using wing's mean aerodynamic chord as a unit of length, let $\langle x_w, z_w\rangle$, $\langle x_{c.g.}, z_{c.g.}\rangle$, $\langle x_l, z_l\rangle$, and $\langle x_p, z_p\rangle$ be the respective dimensionless coordinates of the wing aerodynamic center, vessel's c.g., lines' c.p., and pilot's c.p. (see Fig. 1). Also, let α be the angle of attack, measured between the direction of the flow and the x axis.

The pitching moment coefficient of the vessel about its c.g. is

$$\begin{aligned} C_M = {} & C_{M,w0} + (C_L \cos\alpha + C_{d,w} \sin\alpha)(x_w - x_{c.g.}) \\ & + (C_L \sin\alpha - C_{d,w}\cos\alpha)(z_w - z_{c.g.}) \\ & - C_{d,l}(z_l - z_{c.g.})\cos\alpha + C_{d,l}(x_l - x_{c.g.})\sin\alpha \\ & - C_{d,p}(z_p - z_{c.g.})\cos\alpha + C_{d,p}(x_p - x_{c.g.})\sin\alpha \end{aligned} \tag{1}$$

for $\alpha << 1$ it reduces to

$$\begin{aligned} C_M \approx {} & C_{M,w0} - C_{d,l}(z_l - z_{c.g.}) - C_{d,p}(z_p - z_{c.g.}) \\ & + C_L(x_w - x_{c.g.}) + (C_L\alpha - C_{d,w})(z_w - z_{c.g.}) \end{aligned} \tag{2}$$

It will be assumed that the lift coefficient is linear with the angle of attack, in which case

$$\alpha = (C_L - C_{L,0})/a \tag{3}$$

with a being the lift slope coefficient and $C_{L,0}$ the lift coefficient at zero angle of attack. It will be further assumed that drag coefficients of the lines and pilot are independent of the lift coefficient, whereas the drag coefficient of the wing is given by the parabolic polar

$$C_{d,w} = C_{d,w0} + KC_L^2 \tag{4}$$

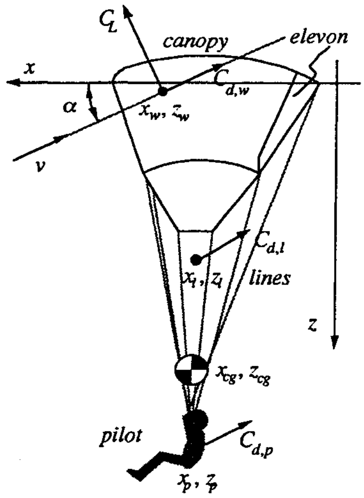

Fig. 1 Notation.

with $C_{d,w0}$ and K constants. Accordingly, Eq. (2) becomes

$$C_M \approx C_{M,0} + [(x_w - x_{c.g.}) - (1/a)C_{L,0}(z_w - z_{c.g.})]C_L + (1/a)(1 - aK)(z_w - z_{c.g.})C_L^2 \tag{5}$$

where

$$C_{M,0} = C_{M,w0} - C_{d,w0}(z_w - z_{c.g.}) - C_{d,l}(z_l - z_{c.g.}) - C_{d,p}(z_p - z_{c.g.}) \tag{6}$$

III. Trim and Stability

If a vessel is to hold its attitude, then

$$C_M = 0 \tag{7}$$

With Eq. (5), Eq. (7) yields a quadratic equation

$$aC_{M,0} + [a(x_w - x_{c.g.}) - C_{L,0}(z_w - z_{c.g.})]C_{L,\text{trim}} + (1 - aK)(z_w - z_{c.g.})C_{L,\text{trim}}^2 = 0 \tag{8}$$

for the lift coefficient $C_{L,\text{trim}}$ at equilibrium (trim). The two obvious solutions of Eq. (8) are

$$C_{L,\text{trim}} = \frac{1}{2(1 - aK)(z_w - z_{c.g.})}\big(-a(x_w - x_{c.g.}) + C_{L,0}(z_w - z_{c.g.}) \pm \{[a(x_w - x_{c.g.}) - C_{L,0}(z_w - z_{c.g.})]^2 - 4aC_{M,0}(1 - aK)(z_w - z_{c.g.})\}^{1/2}\big) \tag{9}$$

Among those two solutions, if they exist, we choose the one at which the vessel is statically stable, i.e., we choose the solution for which

$$\frac{\partial C_M}{\partial C_L} < 0 \quad \text{at} \quad C_L = C_{L,\text{trim}} \tag{10}$$

Assuming, for simplicity, no elastic deformations of the canopy and the lines with the change in lift coefficient, one has that

$$\frac{\partial C_M}{\partial C_L} \approx (x_w - x_{c.g.}) - \frac{1}{a}C_{L,0}(z_w - z_{c.g.}) + \frac{2}{a}(1 - aK)(z_w - z_{c.g.})C_L \tag{11}$$

by Eq. (6). Hence,

$$\frac{\partial C_M}{\partial C_L} \approx \pm\left\{\left[(x_w - x_{c.g.}) - \frac{1}{a}C_{L,0}(z_w - z_{c.g.})\right]^2 - \frac{4}{a}C_{M,0}(1 - aK)(z_w - z_{c.g.})\right\}^{1/2} \tag{12}$$

at $C_L = C_{L,\text{trim}}$, by Eq. (9).

For inequality (10) to hold, the sign in Eq. (12), and correspondingly in Eq. (9), needs to be minus; from which

$$C_{L,\text{trim}} = \frac{1}{2(1 - aK)(z_w - z_{c.g.})}\big(-a(x_w - x_{c.g.}) + C_{L,0}(z_w - z_{c.g.}) - \{[a(x_w - x_{c.g.}) - C_{L,0}(z_w - z_{c.g.})]^2 - 4aC_{M,0}(1 - aK)(z_w - z_{c.g.})\}^{1/2}\big) \tag{13}$$

Note that both $C_{M,0}$ and $C_{L,0}$ are functions of the (generalized) elevons deflection δ_e. With those functions known, Eq. (13) defines $C_{L,\text{trim}}(x_{c.g.}, \delta_e)$.

IV. Forward Center of Gravity Limit

It is clear that solution (13) for $C_{L,\text{trim}}$ exists only if

$$[a(x_w - x_{c.g.}) - C_{L,0}(z_w - z_{c.g.})]^2 \geq 4aC_{M,0}(1 - aK)(z_w - z_{c.g.}) \tag{14}$$

In Eq. (14), the left-hand side (LHS) is nonnegative, whereas the right-hand side (RHS) is either positive or negative, depending on signs of the respective multipliers. It is shown in Appendix A that under normal circumstances $aK < 1$. Also, $z_w - z_{c.g.} < 0$, by the choice of the coordinate system (see Fig. 1). Hence, if $C_{M,0} \geq 0$, then Eq. (14) holds unconditionally, i.e., no direct limitations are imposed on the longitudinal c.g. position. If, on the other hand, $C_{M,0} < 0$, then there exists

$$x_\pm = x_w - (1/a)C_{L,0}(z_w - z_{c.g.}) \pm (1/a)\sqrt{4aC_{M,0}(1 - aK)(z_w - z_{c.g.})} \tag{15}$$

such that Eq. (14) holds if either

$$x_{c.g.} \leq x_- \tag{16}$$

or

$$x_{c.g.} \geq x_+ \tag{17}$$

For $x_{c.g.} = x_\pm$, the absolute value of $C_{L,\text{trim}}$ is

$$C_{L,1} = \sqrt{\frac{aC_{M,0}}{(1 - aK)(z_w - z_{c.g.})}} \tag{18}$$

by Eqs. (13) and (15). Hence, by Eqs. (13) and (18)

$$C_{L,\text{trim}} \geq C_{L,1} \quad \text{for each} \quad x_{c.g.} \leq x_- \tag{19}$$

$$C_{L,\text{trim}} \leq -C_{L,1} \quad \text{for each} \quad x_{c.g.} \geq x_+ \tag{20}$$

Since negative values of $C_{L,\text{trim}}$ are irrelevant in the present discussion, Eq. (17) is ruled out by Eq. (20); whence the c.g. should be positioned behind x_-, by Eq. (16). In this case, Eq. (19) implies that there exists a lower bound on the lift coefficient possibly attainable at trim.

Note that a gliding parachute with $C_{M,0} < 0$ is neutrally stable at $x_{c.g.} = x_-$, by Eqs. (12) and (15). Since the vessel is supposed to be stable at $x_{c.g.} < x_-$, by Eqs. (12), (13), and (16), moving the c.g. backward seems to have a stabilizing effect. This may be formally shown as follows. Substitute Eq. (15) in Eq. (12) so as to obtain

$$\frac{\partial C_M}{\partial C_L} = -\{(x_- - x_{c.g.})[2(x_w - x_-) - \frac{2}{a} C_{L,0}(z_w - z_{c.g.}) + (x_- - x_{c.g.})]\}^{1/2}$$

In this expression $(x_w - x_-) - (1/a)C_{L,0}(z_w - z_{c.g.}) \geq 0$, by Eq. (15); $z_w - z_{c.g.} < 0$, by the choice of the coordinate system; and $C_{L,0}$ is positive for a typical canopy. Hence, an increase in $x_- - x_{c.g.}$ makes the derivative $\partial C_M/\partial C_L$ more negative.

Using typical values for all pertinent parameters appearing on the RHS of Eq. (6), one may find that the sum of drag contributions to $C_{M,0}$ is usually positive (see Appendix B). Hence, at least in principle, by flattening the profile one can design a vessel with positive $C_{M,0}$. As already cited, such a vessel will have no apparent limitations on its longitudinal c.g. position, and therefore, it could be designed to fly at any desired lift coefficient below stall [see Eq. (13)].

At the same time, it seems improbable that one can design an elevons-controlled gliding parachute in such a way that it will have a reasonable range of accessible lift coefficients on the one hand, and nonnegative $C_{M,0}$ for all possible elevons deflections on the other. But in order to trim the vessel with negative $C_{M,0}$, the most forward c.g. position should be limited by Eq. (16). Thus, given the range $\Delta = (\delta_{\min}, \delta_{\max})$ of usable elevons deflections, the requirement that H1 (a trim condition should exist for each δ_e in Δ) limits the forward c.g. position by

$$x_{c.g.,1} = \inf[x_-(\delta_e) | \delta_e \in \Delta \quad \text{and} \quad C_{M,0}(\delta_e) \leq 0] \tag{21}$$

V. Simplified Model

To simplify all matters considerably, let us make the following two assumptions:

A1: $C_{M,0}$ is a decreasing function of δ_e;

In the context of A1, note that $\partial C_{M,0}/\partial\delta_e = \partial C_{M,w0}/\partial\delta_e - (z_w - z_{c.g.})\partial C_{d,w0}/\partial\delta_e$, by Eq. (6). Given that the canopy is not stalled, the first term of the RHS of this equation is always negative. At the same time, the second term, although yielding a positive contribution, vanishes for small elevons deflections (see, e.g., Ref. 2, p. 109). Hence, at least for a certain range of elevons deflections, A1 holds.

A2: $(\partial C_{M,0}/\partial\delta_e)(\partial C_{L,0}/\partial\delta_e)$ is a negative-valued nondecreasing function of δ_e.

In the context of A2, note that in the case where the canopy is approximated by an equivalent rigid straight wing equipped with a plain flap, the ratio $(\partial C_{M,0}/\partial\delta_e)/(\partial C_{L,0}/\partial\delta_e)$ is shown in Sec. VIII to be independent of δ_e for small deflections. Hence, at least under these circumstances, A2 holds.

Under assumption A1, there exists δ_0, such that $C_{M,0}(\delta_0) = 0$. Since no forward limit bounds the c.g. position when $C_{M,0}$ is nonnegative, it will be implicitly assumed that $\delta_0 < \delta_{\max}$.

Let $\delta_1 = \sup(\delta_0, \delta_{\min})$ be the minimal allowable elevons deflection for which $C_{M,0}$ is nonpositive. By Eq. (15), x_- is continuous on $(\delta_1, \delta_{\max})$. Hence, the minimum of x_- on this interval corresponds either to an extremum of x_-, or to one of the endpoints δ_1 and $\delta_{\max}$.

Seeking the extremum of x_-, we need to find such a deflection δ_{ex} of the elevons, that

$$\frac{\partial x_-}{\partial \delta_e} = 0 \quad \text{at} \quad \delta_e = \delta_{ex} \tag{22}$$

Substituting Eq. (15) in Eq. (22), one readily finds that δ_{ex} is a solution of the equation

$$C_{M,0} = \frac{a(1 - aK)}{z_w - z_{c.g.}} \left(\frac{\partial C_{M,0}}{\partial \delta_e}\right)^2 \left(\frac{\partial C_{L,0}}{\partial \delta_e}\right)^{-2} \tag{23}$$

With $z_w < z_{c.g.}$, the expression on the RHS of Eq. (23) is a negative-valued nondecreasing function of δ_e, by assumption A2. Hence, under assumptions A1 and A2, Eq. (23) has a single solution. This solution must satisfy

$$\delta_{ex} > \delta_0 \tag{24}$$

by definition. Moreover, $x_-(\delta_{ex})$ is a minimum, by Eq. (15). Accordingly, in two particular cases where $\delta_{ex} < \delta_{\min}$ and $\delta_{ex} \in \Delta$, one has that $x_{c.g.,1} = x_-(\delta_{\min})$ and $x_{c.g.,1} = x_-(\delta_{ex})$, respectively.

For future reference, note that

$$C_{L,\text{trim}}(x_{c.g.}, \delta_{ex}) \geq \frac{a}{|z_w - z_{c.g.}|} \left|\frac{\partial C_{M,0}}{\partial \delta_e}\right|_{\delta_e = \delta_{ex}} \left|\frac{\partial C_{L,0}}{\partial \delta_e}\right|^{-1}_{\delta_e = \delta_{ex}} \tag{25}$$

for each $x_{c.g.} \leq x_-(\delta_{ex})$ by Eqs. (23), (18), and (19).

VI. Control Derivatives

Consider the derivatives $\partial C_{L,\text{trim}}/\partial\delta_e$ and $\partial C_{L,\text{trim}}/\partial x_{c.g.}$. Differentiate, in turn, on both sides of Eq. (8) with respect to δ_e and $x_{c.g.}$, and use Eq. (8) in the resulting expressions so as to obtain

$$\frac{\partial C_{L,\text{trim}}}{\partial x_{c.g.}} = \frac{aC_{L,\text{trim}}^2}{-aC_{M,0} + (1 - aK)(z_w - z_{c.g.})C_{L,\text{trim}}^2} \tag{26}$$

$$\frac{\partial C_{L,\text{trim}}}{\partial \delta_e} = \frac{(z_w - z_{c.g.})C_{L,\text{trim}}^2 \dfrac{\partial C_{L,0}}{\partial \delta_e} - aC_{L,\text{trim}} \dfrac{\partial C_{M,0}}{\partial \delta_e}}{-aC_{M,0} + (1 - aK)(z_w - z_{c.g.})C_{L,\text{trim}}^2} \tag{27}$$

The expression appearing in the denominator on the RHS of either Eq. (26) or Eq. (27) is negative for each $x_{c.g.} < x_{c.g.,1}$ by Eqs. (18), (19), and (21). Hence,

$$\frac{\partial C_{L,\text{trim}}}{\partial x_{c.g.}} < 0 \quad \text{for each} \quad x_{c.g.} < x_{c.g.,1} \tag{28}$$

That is, moving the c.g. forward reduces the lift coefficient at trim, as in a conventional vessel.

Predictable longitudinal control of a vessel requires that the lift coefficient at trim should be a monotonic function of controls deflection. For a gliding parachute this requirement may be specified as H2 (lift coefficient at trim should be an increasing function of elevons deflection) or, equivalently,

$$\frac{\partial C_{L,\text{trim}}}{\partial \delta_e} > 0 \quad \text{for each} \quad \delta_e \text{ in } \Delta \tag{29}$$

From Eq. (28) it follows that in order to satisfy Eq. (29) for some $x_{c.g.} < x_{c.g.,1}$ one needs

$$(z_w - z_{c.g.})C_{L,\text{trim}} \frac{\partial C_{L,0}}{\partial \delta_e} - a \frac{\partial C_{M,0}}{\partial \delta_e} < 0 \quad \text{for each} \quad \delta_e \text{ in } \Delta \tag{30}$$

In Eq. (30), $\partial C_{L,0}/\partial\delta_e$ is normally positive; whereas $z_w - z_{c.g.}$ is negative. Hence, with

$$C_{L,2} = -\frac{a}{z_{c.g.} - z_w} \left(\frac{\partial C_{M,0}}{\partial \delta_e}\right) \left(\frac{\partial C_{L,0}}{\partial \delta_e}\right)^{-1} \tag{31}$$

Eq. (29) imposes a restriction

$$C_{L,\text{trim}}(x_{\text{c.g.}}, \delta_e) > C_{L,2}(\delta_e) \tag{32}$$

on the lift coefficient that can be obtained for each δ_e in Δ. But if Eq. (29) holds, then $C_{L,\text{trim}}$ is an increasing function of δ_e. At the same time, under assumption A2, $C_{L,2}$ is a nonincreasing function of δ_e. Hence, Eq. (32) can be reduced to

$$C_{L,\text{trim}}(x_{\text{c.g.}}, \delta_{\min}) > C_{L,2}(\delta_{\min}) \tag{33}$$

Equation (33) in conjunction with Eqs. (28) and (8) can be used to define an additional restriction on the most forward c.g. position. With

$$x_{\text{c.g.,trim}}(C_L, \delta_e) = x_w + C_{M,0}(\delta_e)/C_L + (z_{\text{c.g.}} - z_w)[C_{L,0}(\delta_e) - (1 - aK)C_L]/a \tag{34}$$

the longitudinal c.g. position needed to trim a vessel at lift coefficient C_L with elevons deflected at δ_e, this restriction takes on the form

$$x_{\text{c.g.}} < x_{\text{c.g.,trim}}[C_{L,2}(\delta_{\min}), \delta_{\min}] \tag{35}$$

But, Eq. (32) and, hence, Eq. (35), hold identically if $x_{\text{c.g.}} < x_-(\delta_{\text{ex}})$ and $\delta_e \geq \delta_{\text{ex}}$, by Eqs. (25) and (28). Accordingly

$$x_-(\delta_{\min}) < x_{\text{c.g.,trim}}[C_{L,2}(\delta_{\min}), \delta_{\min}] \quad \text{if} \quad \delta_{\text{ex}} < \delta_{\min} \tag{36}$$

$$x_-(\delta_{\text{ex}}) > x_{\text{c.g.,trim}}[C_{L,2}(\delta_{\min}), \delta_{\min}] \quad \text{if} \quad \delta_{\text{ex}} > \delta_{\min} \tag{37}$$

Concurrently, the minimal lift coefficient attainable at trim is $C_{L,1}(\delta_{\min})$ when $\delta_{\text{ex}} < \delta_{\min}$, and $C_{L,2}(\delta_{\min})$ when $\delta_{\text{ex}} > \delta_{\min}$ [see Eqs. (19) and (33), respectively].

VII. Minimal Lift Coefficient at Trim

Concluding the results of the preceding three sections, the forward c.g. limit, and, concurrently, the minimal attainable lift coefficient are consequences of the requirements H1 and H2. From the design point of view, H1 seems to be weaker than H2. In fact, as cited in the discussion of Sec. IV, by flattening the wing's camber, one can design a vessel with positive $C_{M,0}(\delta_{\min})$; in which case $\delta_{\text{ex}} > \delta_0 > \delta_{\min}$, by Eq. (24). Hence, the forward c.g. limit is $x_{\text{c.g.,trim}}[C_{L,2}(\delta_{\min}), \delta_{\min}]$, by Eq. (37), whereas the minimal lift coefficient limit is $C_{L,2}(\delta_{\min})$, by Eq. (33); both limits are consequences of H2.

By Eq. (31), $C_{L,2}(\delta_{\min})$ depends on several design parameters, of which the most noticeable are the relative lengths of the elevons chord and of the lines. An increase in either of these two parameters typically reduces $C_{L,2}$. Increasing the line length relative to the mean aerodynamic chord increases $z_{\text{c.g.}} - z_w$ (see Sec. II), whereas increasing the elevons chord relative to the mean aerodynamic chord reduces the ratio of the derivatives $\partial C_{M,0}/\partial \delta_e$ and $\partial C_{L,0}/\partial \delta_e$.

The last point is best elucidated by considering the limit where the elevons extend over the whole wing. In this case, elevon deflections do not affect $C_{M,0}$ at all; whence $C_{L,2} = 0$. In other words, no limit is imposed on the minimal lift coefficient at trim. Since $C_{L,2}$ is clearly positive in the case where the elevons occupy the trailing-edge area only, the initial assertion follows. In this context also note that application of the all-wing elevons is analogous to the change in c.g. position (relative to the canopy-fixed coordinate system depicted in Fig. 1). Hence, the present conclusion that minimal lift coefficient at trim is unbounded with $C_{M,0} > 0$ and all-wing elevons agree with the comparable conclusion of Sec. IV.

Line length is a fairly inflexible parameter, as it affects most stability derivatives of the vessel. In fact, the ratio between the lines length and wingspan is almost the same in all present designs. With this said, parachutes with high-aspect wings (having short aerodynamic chord) should have higher top speed than those with low-aspect wings. This conclusion is in apparent agreement with present design trends.

Large elevons require large control forces. Also, they may complicate actuators design, as the lines will need to be shortened, in turn, from the trailing edge forward. Yet, they hold the potential of a gliding parachute with aerodynamically unlimited minimal lift coefficient.

VIII. Estimate of $C_{L,2}$

We seek an order-of-magnitude estimate of $C_{L,2}(\delta_{\min})$. Toward this end, approximate the canopy as a straight rigid wing equipped with a trailing-edge full span plain (sealed) flap. Let c_e be the relative chord of the flap and $\delta_e << 1$ be the angle of its deflection. Neglecting in Eq. (6) the dependence of $C_{d,w0}$ on δ_e (Ref. 2, p. 109), one readily finds that

$$\frac{\partial C_{M,0}}{\partial \delta_e} \approx \frac{\partial C_{M,w0}}{\partial \delta_e} \tag{38}$$

In the framework of a lifting-line theory, both $\partial C_{M,w0}/\partial \delta_e$ and $(1/a)\partial C_{L,0}/\partial \delta_e$ are known to be independent of the AR (Ref. 3, pp. 150 and 139, respectively). Hence, using results of the thin airfoil theory (Ref. 3, pp. 90 and 91),

$$C_{L,2} \approx \frac{\pi}{z_{c.g.} - z_w} \frac{\sin \theta_e(1 + \cos \theta_e)}{2(\theta_e + \sin \theta_e)} \tag{39}$$

where

$$\theta_e = \cos^{-1}(1 - 2c_e) \tag{40}$$

With $z_{\text{c.g.}} - z_w$ typically between 2.2–2.5, and relative elevons chord typically between 20–30%, Eq. (39) yields $C_{L,2}$ between 0.57–0.4 (see Fig. 2). At 35 N/m^2 wing loading, these values correspond to a flight speed between 36–43 km/h at standard sea level conditions, in good agreement with the top speed data of current designs.

In this context it is worth noting that in soaring, the high-speed potential is needed mainly to advance (penetrate) against the wind. Accordingly, the lift coefficient $C_{L,p}$ yielding maximal ground (penetration) speed seems to be an ultimate design target for the minimal lift coefficient limit of a gliding parachute. This lift coefficient is shown in Appendix C to be, roughly, $\sqrt{2}C_{D,0}$ where $C_{D,0}$ is the respective zero-lift drag coefficient. For a typical gliding parachute $C_{D,0} \sim 0.08$ (see Appendix B), whence $C_{L,p} \sim 0.12$, corresponding to the top speed of about 70 km/h at standard sea level conditions and 35 N/m^2 wing loading (see Appendix C).

IX. Speed System

It is commonly accepted with parachute designers that $C_{L,\text{trim}}$ at $\delta_e = \delta_{\min}$ is set to be slightly less than the lift coefficient $C_{L,\text{glide}}$ giving the best glide ratio; practice shows that this setting minimizes average pilot workload under typical op-

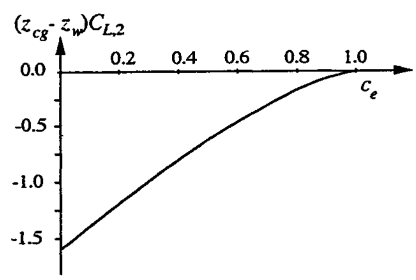

Fig. 2 Minimal lift coefficient limit as a function of elevons relative chord.

erating conditions. In the framework of the present model, where drag coefficient is linear in C_L^2 [see Eq. (4)], it is well known (e.g., Ref. 4, p. 167) that

$$C_{L,\text{glide}} = \sqrt{C_{D,0}/K} \tag{41}$$

For a typical gliding parachute (see Appendix B) Eq. (41) yields $C_{L,\text{glide}} \sim 0.9$. As this figure is much larger than the typical minimal lift coefficient limit of about 0.5 (see Sec. VIII), a pilot cannot utilize the top speed potential of its vessel using elevons only; to fly faster a more forward c.g. position is needed. The latter may be attained, e.g., by pulling the lines attached to the forward half of the canopy (recall that c.g. position is defined relative to the coordinate system depicted in Fig. 1); a system of lines and pulleys designed to do that is commonly referred to as a speed system.

X. Rear Center-of-Gravity Limit

From the previous discussion it follows that no stability (H1) or control (H2) requirements bound the most rear c.g. position. At the same time, for a given elevons deflection, the lift coefficient at trim increases when the c.g. moves backward, by Eq. (28). Consequently, there exists a certain backward c.g. position where the canopy will stall at the maximal (mechanical) elevons deflection. Hence, the requirement that H3 (maximal elevons deflection $\delta_{\max}$ should be attained without stalling the canopy) imposes a restriction on the most rear c.g. position. Specifically, with $C_{L,\max}(\delta_e)$ the maximal lift coefficient at stall with elevons deflected at δ_e, H3 yields

$$x_{\text{c.g.}} < \sup\{x_{\text{c.g.,trim}}[C_{L,\max}(\delta_e), \delta_e] \,|\, \delta_e \in \Delta\} \tag{42}$$

by Eqs. (28) and (34).

Appendix A: Estimate of the Product aK

Assuming that the lift slope a and the drag-polar constant K of an arched wing can be roughly approximated by the values of those constants corresponding to a comparable straight wing, one has that

$$a \approx \frac{a_{2D}\pi A}{a_{2D} + \sqrt{a_{2D}^2 + \pi^2 A^2}} \tag{A1}$$

$$K \approx (1 + \delta + \pi k_s A)/\pi A \tag{A2}$$

(Ref. 2, pp. 137, 194). Here, a_{2D} is the lift-slope coefficient of the wing's profile, A is the (projected) AR of the wing, δ is a planform efficiency correction factor, and k_s is a separation drag correction factor.

With all parameters appearing on the RHSs of Eqs. (A1) and (A2) being nonnegative, it is fairly straightforward to show that the product aK has a maximum at

$$A = \frac{2k_s a_{2D}^2(1 + \delta)}{\pi[(1 + \delta)^2 - k_s^2 a_{2D}^2]} \tag{A3}$$

Hence, if $k_s a_{2D} < 1 + \delta$, then

$$aK \le \frac{k_s^2 a_{2D}^2 + (1 + \delta)^2}{2(1 + \delta)} < 1 \tag{A4}$$

Typically, $a_{2D} < 6$, $\delta > 0$, and $k_s < 0.1$ (see Appendix B); from which Eq. (A4) holds under normal circumstances.

Appendix B: Typical Values of Some Aerodynamic Coefficients

Pertinent aerodynamic coefficients of a typical gliding parachute with a canopy of 26 m^2 and a span of 10 m are estimated next to within one significant digit.

Assume wing section LS1-0417 with 8.4% chord air intake. Assume no aerodynamic interference between the canopy and the lines. Then, $C_{d,w0} \sim 0.03$ and $a_{2D} \sim 5$ by Figs. 7 and 9 of Ref. 5.

To estimate the coefficients a and K, approximate the canopy by an equivalent straight rigid wing of the same projected AR. Then, $a \sim 3.3$ by Eq. (A1); whereas $\delta \sim 0.1$ by Fig. 4.22 of Ref. 2, and $k_s \sim 0.02$ by Fig. 7 of Ref. 5. Consequently, $K \sim 0.1$ by Eq. (A2).

Assume that the canopy is attached to a pilot by 500 m of 1.5-mm lines of a circular cross section. Assume all lines to be almost perpendicular to the flow and no aerodynamic interference between them. Given a typical flight velocity of 10 m/s, the corresponding crossflow Reynolds number on a single line is of the order of 10^3; whence the drag coefficient of all lines, based on their frontal area, should be about 1.1, by Fig. 4.6 of Ref. 2. Accordingly, $C_{d,l} \sim 0.03$.

Assume that a pilot holds a sitting position. Then, the pilot's drag area should be about 0.5 m^2, by Ref. 6; where $C_{d,p} \sim 0.02$. The parasitic drag coefficient $C_{D,0}$ of the entire vessel is $C_{D,0} = C_{d,w0} + C_{d,l} + C_{d,p} \sim 0.08$.

Appendix C: Maximal Penetration Speed of a Gliding Parachute

Consider a vessel gliding without an engine in quiescent air of density ρ. Let W and S be the weight of the vessel and its wing area, respectively. Also, let V be the true airspeed of the vessel and γ be the glide-path-angle. Assuming unaccelerated motion, one has that

$$W \cos\gamma = \tfrac{1}{2}\rho V^2 S C_L \tag{C1}$$

$$W \sin\gamma = \tfrac{1}{2}\rho V^2 S C_D \tag{C2}$$

where C_L and C_D are the lift and drag coefficients, respectively. The horizontal velocity component (penetration speed) V_H of the vessel is, by definition,

$$V_H = V\cos\gamma \tag{C3}$$

From Eqs. (C1) and (C2)

$$\cos\gamma = C_L/\sqrt{C_L^2 + C_D^2} \tag{C4}$$

from which

$$V_H^2 = V^2\cos^2\gamma = \frac{2WC_L^2}{\rho S(C_L^2 + C_D^2)^{3/2}} \tag{C5}$$

by Eq. (C1).

Noting that $V_H \to 0$ as either $C_L \to 0$ or $C_L \to \infty$, V_H has an extremum. Accordingly, the lift coefficient maximizing V_H will be found from the equation

$$2C_D^2 = C_L^2 + 3C_L C_D\left(\frac{\partial C_D}{\partial C_L}\right) \tag{C6}$$

With parabolic drag polar

$$C_D = C_{D,0} + KC_L^2 \tag{C7}$$

Eq. (C6) becomes

$$4KC_D^2 + (1 - 6KC_{D,0})C_D - C_{D,0} = 0 \tag{C8}$$

Equation (C8) has an obvious solution

$$C_D = (1/8K)[-1 + 6KC_{D,0} + (1 + 4KC_{D,0} + 36K^2C_{D,0}^2)^{1/2}] \tag{C9}$$

from which

$$C_L = (1/2K\sqrt{2})[(1 + 4KC_{D,0} + 36K^2C_{D,0}^2)^{1/2} - 1 - 2KC_{D,0}]^{1/2} = \sqrt{2}C_{D,0}[1 + \mathcal{O}(KC_{D,0})] \tag{C10}$$

by Eq. (C7). At this lift coefficient

$$\cos\gamma = \sqrt{\tfrac{2}{3}}[1 + \mathcal{O}(KC_{D,0})] \tag{C11}$$

$$V = \sqrt{(2W/\sqrt{3}\rho SC_{D,0})}[1 + \mathcal{O}(KC_{D,0})] \tag{C12}$$

$$V_H = \sqrt{(4W/3\sqrt{3}\rho SC_{D,0})}[1 + \mathcal{O}(KC_{D,0})] \tag{C13}$$

by Eqs. (C4), (C1), and (C3), respectively. Typically, $K \sim 0.1$ and $C_{D,0} \sim 0.08$ (Appendix B). Hence, $C_L \sim 0.12$, by Eq. (C10). With a wing loading of 35 N/m^2, this lift coefficient corresponds to $V \sim 70$ km/h and $V_H \sim 60$ km/h at standard sea level conditions.

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